

Additional information # 1: more on the evolution

s	p	n	s	p	n	$n^*$	s	p	n	$n^*$	s	p	n	$n^*$
1	2	1	43	191	14	1	85	439	354	29	127	709	2,008	139
2	3	1	44	193	24	2	86	443	354	31	128	719	2,141	145
3	5	1	45	197	24	3	87	449	365	32	129	727	2,136	151
4	7	1	46	199	28	5	88	457	369	33	130	733	2,241	159
5	11	2	47	211	30	5	89	461	358	33	131	739	2,354	166
6	13	2	48	223	36	5	90	463	387	34	132	743	2,442	179
7	17	3	49	227	49	5	91	467	413	35	133	751	2,548	181
8	19	3	50	229	44	5	92	479	426	36	134	757	2,649	187
9	23	4	51	233	52	5	93	487	446	37	135	761	2,791	196
10	29	6	52	239	53	5	94	491	454	37	136	769	2,998	203
11	31	4	53	241	55	5	95	499	491	37	137	773	3,017	211
12	37	5	54	251	67	6	96	503	456	38	138	787	3,148	218
13	41	5	55	257	69	6	97	509	490	38	139	797	3,132	228
14	43	9	56	263	72	7	98	521	559	41	140	809	3,385	235
15	47	11	57	269	81	7	99	523	573	43	141	811	3,537	251
16	53	10	58	271	79	7	100	541	594	47	142	821	3,698	256
17	59	12	59	277	85	8	101	547	652	47	143	823	3,914	267
18	61	8	60	281	83	8	102	557	718	50	144	827	4,175	272
19	67	6	61	283	93	8	103	563	757	50	145	829	4,345	279
20	71	11	62	293	81	9	104	569	786	53	146	839	4,360	290
21	73	5	63	307	81	9	105	571	835	55	147	853	4,537	298
22	79	4	64	311	67	9	106	577	839	57	148	857	4,722	310
23	83	6	65	313	66	11	107	587	854	59	149	859	4,862	314
24	89	3	66	317	74	11	108	593	906	63	150	863	4,951	327
25	97	2	67	331	91	12	109	599	916	65	151	877	5,099	340
26	101	1	68	337	88	12	110	601	998	69	152	881	5,228	354
27	103	3	69	347	90	14	111	607	988	72	153	883	5,334	366
28	107	1	70	349	95	15	112	613	1,016	75	154	887	5,460	373
29	109	1	71	353	102	16	113	617	1,078	79	155	907	5,715	388
30	113	3	72	359	126	18	114	619	1,165	82	156	911	5,971	398
31	127	2	73	367	152	19	115	631	1,237	85	157	919	6,366	413
32	131	5	74	373	154	19	116	641	1,295	87	158	929	6,582	423
33	137	6	75	379	166	19	117	643	1,371	91	159	937	6,799	443
34	139	12	76	383	187	20	118	647	1,399	94	160	941	7,006	465
35	149	21	77	389	214	20	119	653	1,523	98	161	947	7,151	474
36	151	19	78	397	206	21	120	659	1,618	102	162	953	7,599	485
37	157	15	79	401	201	21	121	661	1,701	106	163	967	7,920	505
38	163	16	80	409	220	21	122	673	1,773	109	164	971	8,276	523
39	167	24	81	419	241	23	123	677	1,801	117	165	977	8,708	537
40	173	18	82	421	249	25	124	683	1,889	122	166	983	9,086	549
41	179	18	83	431	269	25	125	691	1,885	128	167	991	9,577	567
42	181	17	84	433	320	27	126	701	1,944	133	168	997	9,736	584

Table 7a. Data for  $s \leq 168$ , using BPSW-pseudoprimalty for  $s \geq 110$ . Stage 139 is the last known point where the number of primes decreases. (Interestingly enough, just before that happens,  $n = 4 \cdot p$ , which is the only known – but certainly not the only – example where  $n$  is a multiple of  $p$ .) As for the column  $n^*$ , see Table 7b below.

s	p	n	<i>n*</i>	s	p	n	<i>n*</i>	s	p	n	<i>n*</i>
169	1,009	<b>10,329</b>	601	211	1,297	<b>36,546</b>	2,175	253	1,607	<b>124,407</b>	8,454
170	1,013	<b>10,722</b>	610	212	1,301	<b>37,363</b>	2,236	254	1,609	<b>126,382</b>	8,764
171	1,019	<b>11,048</b>	627	213	1,303	<b>38,018</b>	2,313	255	1,613	<b>128,949</b>	9,084
172	1,021	<b>11,387</b>	655	214	1,307	<b>38,936</b>	2,385	256	1,619	<b>130,732</b>	9,386
173	1,031	<b>11,769</b>	685	215	1,319	<b>39,633</b>	2,484	257	1,621	<b>132,088</b>	9,737
174	1,033	<b>12,251</b>	703	216	1,321	<b>40,152</b>	2,571	258	1,627	<b>133,678</b>	10,114
175	1,039	<b>12,490</b>	730	217	1,327	<b>40,822</b>	2,664	259	1,637	<b>135,308</b>	10,470
176	1,049	<b>12,905</b>	752	218	1,361	<b>42,524</b>	2,761	260	1,657	<b>137,970</b>	10,900
177	1,051	<b>13,209</b>	771	219	1,367	<b>44,138</b>	2,853	261	1,663	<b>141,747</b>	11,343
178	1,061	<b>13,556</b>	797	220	1,373	<b>45,674</b>	2,948	262	1,667	<b>144,129</b>	11,793
179	1,063	<b>13,918</b>	821	221	1,381	<b>47,308</b>	3,049	263	1,669	<b>146,278</b>	12,255
180	1,069	<b>14,393</b>	846	222	1,399	<b>49,203</b>	3,148	264	1,693	<b>150,011</b>	12,788
181	1,087	<b>15,006</b>	877	223	1,409	<b>50,999</b>	3,245	265	1,697	<b>153,719</b>	13,352
182	1,091	<b>15,472</b>	900	224	1,423	<b>53,578</b>	3,344	266	1,699	<b>157,438</b>	13,897
183	1,093	<b>15,695</b>	937	225	1,427	<b>56,009</b>	3,446	267	1,709	<b>160,935</b>	14,542
184	1,097	<b>16,075</b>	967	226	1,429	<b>58,245</b>	3,546	268	1,721	<b>164,713</b>	15,257
185	1,103	<b>16,548</b>	1,002	227	1,433	<b>60,614</b>	3,686	269	1,723	<b>168,209</b>	16,035
186	1,109	<b>16,852</b>	1,043	228	1,439	<b>63,012</b>	3,812	270	1,733	<b>171,914</b>	16,847
187	1,117	<b>17,548</b>	1,068	229	1,447	<b>65,622</b>	3,906	271	1,741	<b>175,208</b>	17,721
188	1,123	<b>17,926</b>	1,102	230	1,451	<b>67,609</b>	4,048	272	1,747	<b>178,924</b>	18,702
189	1,129	<b>18,415</b>	1,131	231	1,453	<b>69,411</b>	4,176	273	1,753	<b>182,442</b>	19,771
190	1,151	<b>19,145</b>	1,167	232	1,459	<b>71,677</b>	4,298	274	1,759	<b>186,286</b>	20,973
191	1,153	<b>19,690</b>	1,193	233	1,471	<b>73,978</b>	4,436	275	1,777	<b>190,466</b>	22,336
192	1,163	<b>20,364</b>	1,229	234	1,481	<b>76,816</b>	4,602	276	1,783	<b>196,294</b>	23,692
193	1,171	<b>21,154</b>	1,266	235	1,483	<b>78,753</b>	4,721	277	1,787	<b>200,525</b>	25,245
194	1,181	<b>21,954</b>	1,314	236	1,487	<b>81,051</b>	4,866	278	1,789	<b>204,944</b>	26,932
195	1,187	<b>22,688</b>	1,339	237	1,489	<b>82,647</b>	5,031	279	1,801	<b>209,420</b>	28,802
196	1,193	<b>23,528</b>	1,388	238	1,493	<b>84,106</b>	5,209	280	1,811	<b>214,672</b>	30,918
197	1,201	<b>24,295</b>	1,420	239	1,499	<b>85,756</b>	5,395	281	1,823	<b>218,990</b>	33,330
198	1,213	<b>25,085</b>	1,460	240	1,511	<b>87,469</b>	5,558	282	1,831	<b>224,272</b>	36,167
199	1,217	<b>25,717</b>	1,506	241	1,523	<b>90,333</b>	5,745	283	1,847	<b>230,486</b>	39,373
200	1,223	<b>26,680</b>	1,561	242	1,531	<b>93,284</b>	5,906	284	1,861	<b>238,339</b>	43,162
201	1,229	<b>27,569</b>	1,600	243	1,543	<b>96,279</b>	6,095	285	1,867	<b>246,127</b>	47,571
202	1,231	<b>28,242</b>	1,651	244	1,549	<b>99,150</b>	6,291	286	1,871	<b>253,236</b>	52,756
203	1,237	<b>28,917</b>	1,698	245	1,553	<b>101,723</b>	6,510	287	1,873	<b>259,946</b>	59,252
204	1,249	<b>29,871</b>	1,748	246	1,559	<b>104,560</b>	6,707	288	1,877	<b>266,707</b>	67,509
205	1,259	<b>30,707</b>	1,800	247	1,567	<b>107,041</b>	6,914	289	1,879	<b>272,114</b>	77,932
206	1,277	<b>31,555</b>	1,850	248	1,571	<b>109,795</b>	7,142	290	1,889	<b>278,512</b>	91,869
207	1,279	<b>32,633</b>	1,911	249	1,579	<b>111,912</b>	7,364	291	1,901	<b>285,821</b>	111,253
208	1,283	<b>33,494</b>	1,986	250	1,583	<b>114,508</b>	7,624	292	1,907	<b>292,121</b>	140,561
209	1,289	<b>34,556</b>	2,043	251	1,597	<b>117,842</b>	7,904	293	1,913	<b>298,404</b>	191,550
210	1,291	<b>35,744</b>	2,109	252	1,601	<b>121,087</b>	8,176	294	1,931	<b>305,779</b>	305,779

Table 7b. Data for  $169 \leq s \leq 294$ .  $n^*$  stands for the number of surviving branches, backtracked from the primes of stage 294. This number will be subject to change for  $s$  larger than  $\approx 130$  by the time the calculation is taken further since many of them will get cancelled out, so this column only gives a momentary picture. Nevertheless, the rate of decrease for larger  $s$  may be of interest in itself.

It's also a question of probability to tell when  $n^*$  should have reached a lower limit, for example  $n^* = 65$  at  $s = 109$  seems to have settled after  $s = 277$ .

The calculation up to the point given above takes about two months with Pari/GP [5] on a single CPU core of a state-of-the-art PC with the following program, which in the given form starts at  $s = 29$  and keeps track of the numbers as  $a = q_{s,1}$  and a memory-friendly vector  $d$  consisting of only the consecutive differences between the numbers of a given stage where  $d_1 = 0$  and, for  $i > 1$ ,  $d_i = q_{s,i} - q_{s,i-1}$ :

```

{
a=350842542483891235293716663559065020274899073;
d=[0]; \\ a and d can also be read in from previously calculated data

s=0;
b=a;
while(b>1, s++; b\=prime(s));
i=#d;
gettime();
while(1,
  s++;
  p=prime(s);
  o=a*p;
  c=d;
  e=floor(exp(sqrt(3*s)/2)/7);
  d=vector(e);
  m=i;
  i=0;
  for(j=1, m,
    o+=c[j]*p;
    y=vector(p);
    forprime(b=3, p-2,
      r=b-lift(Mod(o, b));
      forstep(l=r, p, b, y[l]=1)
    );
    forprime(b=p+2, floor(p^1.9),
      r=b-lift(Mod(o, b));
      if(r<p, y[r]=1)
    );
    forstep(k=2, p-1, 2,
      if(!y[k],
        q=o+k;
        if(ispseudoprime(q),
          i++;
          if(i>1, d[i]=q-z, a=q);
          z=q
        )
      )
    );
  );
g=floor(gettime()/1000);
x="[";
f=floor(g/3600); if(f, x=Str(x, f"h "));
f=floor(g/60); if(f, x=Str(x, f"%60m "));
x=Str(x, g%60"s");
t=Str("Level "p);
print(t": "i" possibilities "x);
t=Str("p#Y "t".txt");
write(t, "a="a"; d="vecextract(d, Str("1.."i)))
)
}

```

Be sure to set `allocatemem(5*10^7)` or higher when aiming for  $s = 300$  or beyond.